# I. Zeeman Effect - Revisited

- No external $\vec{B}$-field ($\vec{B}_{ext} = 0$), we have $\vec{B}_{int}$ that leads to spin-orbit interaction (thus $\hat{H}'_{so}$) [always there!]
- Zeeman Effect: There is an externally applied $\vec{B}$-field ($\vec{B}_{ext} \neq 0$)

  Additional interaction energies (terms) in Hamiltonian

  - $\vec{L} \rightarrow \vec{\mu}_L = \frac{-e}{2m_e}\vec{L}$ leads to $\left(-\vec{\mu}_L \cdot \vec{B}_{ext}\right)$ ← considered in "Normal Zeeman effect"
  - $\vec{S} \rightarrow \vec{\mu}_S = \frac{-e}{m_e}\vec{S}$ leads to $\left(-\vec{\mu}_S \cdot \vec{B}_{ext}\right)$

  ↑ new terms in Hamiltonian when $\vec{B}_{ext} \neq 0$

Cannot avoid it as $\vec{S}$ is already invoked in $\hat{H}_{so}$ (spin-orbit interaction)

$$\hat{H}'_{Zeeman} = \hat{H}'_z = -\hat{\vec{\mu}}_L \cdot \vec{B}_{ext} - \hat{\vec{\mu}}_S \cdot \vec{B}_{ext} = -\hat{\vec{\mu}}_{total} \cdot \vec{B}_{ext}$$

$$= \frac{e}{2m_e}\hat{\vec{L}} \cdot \vec{B}_{ext} + \frac{e}{m_e}\hat{\vec{S}} \cdot \vec{B}_{ext} \qquad \left[\vec{\mu}_{total} = \vec{\mu}_L + \vec{\mu}_S\right]$$

$$= \frac{e}{2m_e}\left(\hat{\vec{L}} + 2\hat{\vec{S}}\right) \cdot \vec{B}_{ext} \quad (35)$$

"Zeeman term" that goes into Hamiltonian

[Note: $\vec{L} + 2\vec{S} \neq \vec{J}$ as $\vec{J} = \vec{L} + \vec{S}$]

- Atom in $\vec{B}_{ext}$: The full QM problem is to solve TISE with

$$\boxed{\hat{H} = \hat{H}_{atom} + \underbrace{\hat{H}'_{so}}_{\text{spin-orbit interaction } f(r)\,\vec{S}\cdot\vec{L} \text{ [independent of } \vec{B}_{ext}\text{]}} + \underbrace{\hat{H}'_z}_{\text{Zeeman } \frac{e}{2m_e}(\vec{L}+2\vec{S})\cdot\vec{B}_{ext}} \left(+ \hat{H}'_{rel}\right)} \quad (36)$$

What to do?

$\hat{H}'_{rel}$: even a relativistic correction [ignore here!] [less important in atoms other than hydrogen]

Think like a physicist! $\hat{H} = \hat{H}_{atom} + \underbrace{\hat{H}'_{so} + \hat{H}'_z}$

- Two things to do? Do the more important thing (part) first and do it more seriously (rigorously)!
- Which term is more important?
  - No standard answer!
  - Depends on situation, i.e. $\vec{B}_{ext}$

## The Big Picture

- Strong $\vec{B}_{ext}$ field: $\hat{H} = \underbrace{(\hat{H}_{atom} + \hat{H}'_z)}_{\text{handle more accurately}} + \underbrace{\hat{H}'_{so}}_{\text{perturbation}}$ (37a)

  ($\hat{H}_z \propto \vec{B}_{ext}$) more important

- Weak $\vec{B}_{ext}$ field: $\hat{H} = \underbrace{(\hat{H}_{atom} + \hat{H}'_{so})}_{\text{treated under spin-orbit interaction}} + \overbrace{\hat{H}'_z}$ (37b)

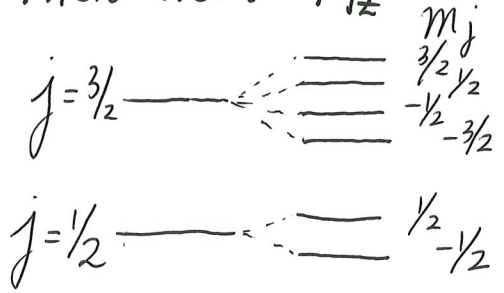
  $\underbrace{B_{int}}_{\hat{H}'_{so}}$ more important (order ~ 1 Tesla)

## Strong $\vec{B}_{ext}$

- Treat $\hat{H}'_z = \frac{eB_{ext}}{2m}(\hat{L}_z + 2\hat{S}_z)$ first
  - states labelled by $(n, \ell, m_\ell, s, m_s)$ are eigenstates of $(\hat{H}_{atom} + \hat{H}'_z)$
  - $\hat{H}'_z$ removes degeneracies behind $m_\ell$ and $m_s$
- With $\psi_{n\ell m_\ell s m_s}$, treat $\hat{H}'_{so}$ as perturbation

## Weak $\vec{B}_{ext}$

- Treat $\hat{H}'_{so} = f(r)\,\vec{S}\cdot\vec{L}$ first
  - states labelled by $(n, \ell, s, j, m_j)$ are good for $\hat{H}'_{so}$
- Then treat $\hat{H}'_z$




[due to $\hat{H}'_{so}$] ↑ [due to $\hat{H}'_z$]

[It is $|\vec{B}_{ext}|$ ($\hat{H}'_z$) vs $|\vec{B}_{int}|$ ($\hat{H}'_{so}$) that decides which term is more important ($\vec{B}_{int} \sim 1$ Tesla)]

Keep this Big Picture in mind as we fill in the math.

## (a) Strong Field Zeeman Effect

$$\hat{H}_{atom}\,\psi_{n\ell m_\ell s m_s} = E_{n\ell}\,\psi_{n\ell m_\ell s m_s}$$

($s = 1/2$ always; for $U(r)$ [spherically symmetric, e.g. atoms]; [$0^{th}$ order problem])

for H-atom, becomes $E_n = \dfrac{-13.6}{n^2}$ eV ("n only" for H-atom)

- The same $\{\psi_{n\ell m_\ell(s) m_s}\}$ also solve $(\hat{H}_{atom} + \hat{H}'_z)$:

$\because$ $\hat{H}'_z\,\psi_{n\ell m_\ell(s) m_s} = \dfrac{e}{2m} B_{ext}\left(\hat{L}_z + 2\hat{S}_z\right)\psi_{n\ell m_\ell(s) m_s} = \dfrac{e\hbar}{2m_e} B_{ext}\,(m_\ell + 2m_s)\,\psi_{n\ell m_\ell(s) m_s}$

($\hat{L}_z$ picks up $(m_\ell\hbar)$; $2\hat{S}_z$ picks up $(m_s\hbar)$; $(m_\ell + 2m_s)$ is the eigenvalue of $\hat{H}'_z$)

$\therefore$ $\left(\hat{H}_{atom} + \hat{H}'_z\right)\psi_{n\ell m_\ell(s) m_s} = \left[\dfrac{-13.6}{n^2} + \mu_B B_{ext}\,(m_\ell + 2m_s)\right]\psi_{n\ell m_\ell(s) m_s}$ (38)

Solved $(\hat{H}_{atom} + \hat{H}'_z)$ exactly!

$\dfrac{-13.6}{n^2}$: for H-atom [removed degeneracies behind $m_\ell$ and $m_s$]

$\mu_B B_{ext}\,(m_\ell + 2m_s)$: eigenvalues now depend on $m_\ell$, $m_s$

Aside:

Recall $\psi_{n\ell m_\ell (s) m_s} = \underbrace{\psi_{n\ell m_\ell}(r,\theta,\phi)}_{R_{n\ell}(r)\cdot Y_{\ell m_\ell}(\theta,\phi)} \cdot \begin{cases} \alpha_z & (\text{if } m_s = +1/2) \text{ "up"} \\ \beta_z & (\text{if } m_s = -1/2) \text{ "down"} \end{cases}$

$\therefore \hat{L}_z \psi_{n\ell m_\ell} = m_\ell \hbar \, \psi_{n\ell m_\ell}$ ($\because Y_{\ell m_\ell}(\theta,\phi)$ is eigenstate of $\hat{L}_z$ with eigenvalue $m\hbar$)

Similarly, $\hat{S}_z \alpha_z = \underbrace{\frac{+\hbar}{2}}_{m_s \hbar \; (m_s = +1/2)} \alpha_z \; ; \; \hat{S}_z \beta_z = \underbrace{-\frac{\hbar}{2}}_{m_s \hbar \; (m_s = -1/2)} \beta_z$

$\therefore$ Eq. (38) follows.

$$\hat{H} = \underbrace{(\hat{H}_{atom} + \hat{H}_z')}_{\text{treated exactly (plays the role of } \hat{H}_0)} + \hat{H}_{so}' \leftarrow \text{treat it perturbatively}$$

- Apply 1st order perturbation theory:

$$E_{so}^{(1)} = \int \psi_{n\ell m_\ell(s) m_s}^{*} \left(f(r)\, \hat{\vec{S}}\cdot\hat{\vec{L}}\right) \psi_{n\ell m_\ell(s) m_s}\, d\tau$$

$$\hat{\vec{S}}\cdot\hat{\vec{L}} = \hat{S}_x\hat{L}_x + \hat{S}_y\hat{L}_y + \hat{S}_z\hat{L}_z$$

$$= \langle f(r)\rangle \left[ \langle \hat{S}_x \rangle^{0^+}\langle \hat{L}_x \rangle^{0^+} + \langle \hat{S}_y \rangle^{0}\langle \hat{L}_y \rangle^{0} + \langle \hat{S}_z\rangle\langle \hat{L}_z\rangle \right]$$

$\langle f(r)\rangle \sim \int |R_{n\ell}(r)|^2 f(r)\, r^2 dr$ (depends on $n, \ell$)

$\langle \hat{S}_x \rangle^{0^+}$: expectation value over spin state specified by $m_s$

$\langle \hat{L}_y \rangle^{0}$: expectation value over orbital AM state specified by $\ell, m_\ell$

$\langle \hat{S}_z\rangle \to m_s\hbar$, $\langle \hat{L}_z\rangle \to m_\ell\hbar$

$$= m_\ell m_s \hbar^2 \langle f(r)\rangle \qquad \text{(Key result)} \qquad (39)$$

$^+$ See next page on handling the spin part, e.g. $\psi_{n\ell m_\ell(s)\,1/2} = \psi_{n\ell m_\ell}(r,\theta,\phi)\cdot\alpha_z$

**Remarks:** Zooming into $\langle \hat{S}_x \rangle = 0$ ← Why?

$\psi_{n\ell m_\ell (s) m_s}$ ← related to spin

$|s\ m_s\rangle = \begin{cases} |\tfrac{1}{2}\ \tfrac{1}{2}\rangle \text{ for } S_z = \frac{\hbar}{2} \text{ (up) state} & \leftarrow \text{also called } \alpha_z \\ |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle \text{ for } S_z = -\frac{\hbar}{2} \text{ (down) state} & \leftarrow \text{also called } \beta_z \end{cases}$

$\therefore \langle \hat{S}_x \rangle$ is either $\langle \alpha_z | \hat{S}_x | \alpha_z \rangle$ or $\langle \beta_z | \hat{S}_x | \beta_z \rangle$

- "States of definite $S_z$ have completely uncertain $S_x$" ($\because [\hat{S}_z, \hat{S}_x] \neq 0$)
  $\therefore \langle \hat{S}_x \rangle = 0$ (by argument)
- More explicitly, $\langle \alpha_z | \hat{S}_x | \alpha_z \rangle = (1\ \ 0)\frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0$ (by math)

  $\langle \beta_z | \hat{S}_x | \beta_z \rangle = (0\ \ 1)\frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0$
- $\langle \hat{L}_x \rangle = \int Y^*_{\ell m_\ell}(\theta, \phi)\, \hat{L}_x\, Y_{\ell m_\ell}(\theta, \phi)\, d\Omega = 0$ (same argument)
- Similar consideration for $\langle \hat{S}_y \rangle$ and $\langle \hat{L}_y \rangle$

# Putting results together (Strong-field):

$$E_{n\ell m_\ell(s) m_s} \approx \underbrace{-\frac{13.6}{n^2}}_{\text{ignored everything (QM I) (H-atom)}} + \underbrace{\mu_B B_{ext}(m_\ell + 2m_s)}_{\text{due to } \hat{H}'_z \text{ (thus } B_{ext}\text{) treated exactly}} + \underbrace{m_\ell m_s \hbar^2 \langle f(r) \rangle}_{\text{due to } \hat{H}'_{so} \text{ 1st order perturbation}} \quad (40)$$

(Done!)

Example: $\ell = 1$ states ($p$ states) $\quad m_\ell = \begin{cases} 1 \\ 0 \\ -1 \end{cases}$ and $m_s = \begin{cases} +\frac{1}{2} \text{ (up)} \\ -\frac{1}{2} \text{ (down)} \end{cases}$ total of 6 states

- ignore everything (1st term in Eq. (34)): $-\frac{13.6}{n^2}$ ——————— $(\ell = 1)(6 \text{ states})$

6 states have

| $m_\ell$ | $m_s$ | $m_\ell + 2m_s$ * | $m_\ell m_s$ + |
|---|---|---|---|
| 1 | $+\frac{1}{2}$ | 2 | $+\frac{1}{2}$ |
| 0 | $+\frac{1}{2}$ | 1 | 0 |
| 1 | $-\frac{1}{2}$ | 0 | $-\frac{1}{2}$ |
| −1 | $+\frac{1}{2}$ | 0 | $-\frac{1}{2}$ |
| 0 | $-\frac{1}{2}$ | −1 | 0 |
| −1 | $-\frac{1}{2}$ | −2 | $+\frac{1}{2}$ |

* Useful for $\hat{H}'_z$ term

\+ Useful for $\hat{H}'_{so}$ term (See Eq. (40))

ignore everything

$\hat{H}_{atom}$ only

$\hat{H}_{atom} + \hat{H}'_z$ | $m_e$ $m_s$ | $-\frac{13.6}{n^2} + \mu_B B_{ext}(m_e + 2m_s)$ | $(\hat{H}_{atom} + \hat{H}'_z) + \hat{H}'_{so}$ | further shift $m_e m_s \hbar^2 \langle f(r) \rangle$

$(1, +\frac{1}{2})$ $-\frac{13.6}{n^2} + 2\mu_B B_{ext}$ ——— ↓↑ $\delta E_{so}$ up $(m_e m_s = +\frac{1}{2})$

$\mu_B B_{ext}$

$(0, +\frac{1}{2})$ $-\frac{13.6}{n^2} + \mu_B B_{ext}$ ——— no shift $(m_e m_s = 0)$

$-\frac{13.6}{n^2}$ ——— $(\ell = 1)$ (6 states)

$(1, -\frac{1}{2})$ $(-1, +\frac{1}{2})$ $-\frac{13.6}{n^2}$ (2 states) ——— ↓↑ $\delta E_{so}$ down $(m_e m_s = -\frac{1}{2})$

$(0, -\frac{1}{2})$ $-\frac{13.6}{n^2} - \mu_B B_{ext}$ ——— no shift $(m_e m_s = 0)$

$(-1, -\frac{1}{2})$ $-\frac{13.6}{n^2} - 2\mu_B B_{ext}$ ——— ↓↑ $\delta E_{so}$ up $(m_e m_s = +\frac{1}{2})$

**Strong Field**

[Split the p states by strong $B_{ext}$ into 5 levels]

(cf. without spin, split into 3 states only (normal Zeeman Effect))

where $\delta E_{so} = \frac{1}{2}\hbar^2 \langle f(r) \rangle$ — tiny shifts due to $\hat{H}'_{so}$

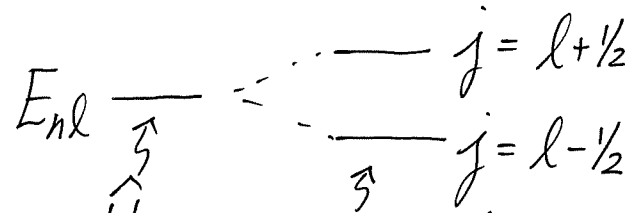
- In any case, the key message is that degenerate states are split in $\vec{B}_{ext}$.
- Similar consideration for other $\ell$.
- Transitions [e.g. $2p \to 1s$] lead to Zeeman Effect (splitting of spectral line in $\vec{B}_{ext}$)

## (b) Weak Field Zeeman Effect

- $\hat{H}'_{so}$ is more important than $\hat{H}'_z$ [recall: $|B_{int}| \sim$ Tesla leads to spin-orbit interaction]
- Do $(\hat{H}_{atom} + \hat{H}'_{so})$ first [Done! see spin-orbit interaction]

use states $\psi_{n\ell(s)jm_j}$ → for $s=1/2$, $\ell \neq 0$: $j = \ell + 1/2,\ \ell - 1/2$
→ for $s=1/2$, $\ell = 0$: $j = 1/2$ only

then




$(j, m_j)$ refer to total angular momentum $\vec{J} = \vec{L} + \vec{S}$

∴ $\hat{H}'_{so}$ makes $\vec{J}$ important (not $L_z$, not $S_z$, they lost their identities)

$\psi_{n\ell(s)jm_j}$ play the role of the "unperturbed" states

- Treat $\hat{H}_Z' = -\hat{\vec{\mu}}_L \cdot \vec{B}_{ext} - \hat{\vec{\mu}}_S \cdot \vec{B}_{ext} = -\underbrace{\hat{\vec{\mu}}_{total}} \cdot \vec{B}_{ext} = \frac{e}{2m_e} \vec{B}_{ext} \cdot (\hat{\vec{L}} + 2\hat{\vec{S}})$

  by 1st order perturbation theory $\quad (\vec{\mu}_L + \vec{\mu}_S)$ = total magnetic moment

$$E^{(1)}_{Zeeman} = \langle n\, l\,(s)\, j\, m_j \,|\, -\hat{\vec{\mu}}_{total} \cdot \vec{B}_{ext} \,|\, n\, l\,(s)\, j\, m_j \rangle$$

$$= \langle n\, l\,(s)\, j\, m_j \,|\, \frac{e}{2m_e} \vec{B}_{ext} \cdot (\hat{\vec{L}} + 2\hat{\vec{S}}) \,|\, n\, l\,(s)\, j\, m_j \rangle \qquad (41)$$

- <u>Need approximation to do (41)</u>:

<u>Why?</u> $(j\ m_j)$ refer to $\vec{J}$, but $\vec{L} + 2\vec{S} = \vec{J} + \vec{S} \neq \vec{J}$

OR $\vec{\mu}_{total} \neq g \left(\frac{-e}{2m_e}\right) \vec{J}$ <u>(strictly speaking)</u>

($g$: some number)

formally, $\vec{\mu}_{total}$ is <u>NOT</u> anti-parallel to $\vec{J}$ as in $\begin{cases} \vec{\mu}_L = (1) \cdot \frac{-e}{2m_e} \vec{L} \\ \vec{\mu}_S = (2) \cdot \frac{-e}{2m_e} \vec{S} \end{cases}$

## Find a way out?

Approximate $\vec{\mu}_{total}$ (not $\propto -\vec{J}$) by $\boxed{\vec{\mu}_{J(approx)} \propto -\vec{J}}$

How? Take the projection of $\vec{\mu}_{total}$ along $\vec{J}$

i.e. project $\underbrace{(\vec{L}+2\vec{S})}$ along $\vec{J}$ $\quad (\because \vec{\mu}_{total} = \frac{-e}{2m_e}(\vec{L}+2\vec{S}))$

$\vec{L}+\vec{S}+\vec{S} = \vec{J}+\vec{S}$ ← project $\vec{S}$ along $\vec{J}$

↑ already along $\vec{J}$ (OK)

- Approximate $\vec{S}$ as a vector along direction of $\vec{J}$ by projecting $\vec{S}$ along $\vec{J}$:

↳ $\left(\vec{S}\cdot\frac{\vec{J}}{|J|}\right)\frac{\vec{J}}{|J|} = \frac{(\vec{S}\cdot\vec{J})}{J^2}\vec{J} = \frac{J^2-L^2+S^2}{2J^2}\vec{J}$

↑ unit vector in direction of $\vec{J}$

$(\because \vec{L} = \vec{J}-\vec{S} \Rightarrow L^2 = J^2+S^2-2\vec{S}\cdot\vec{J})$

$$\therefore\ \vec{L}+2\vec{S} \underset{\text{approximation}}{\approx} \left(1+\frac{J^2-L^2+S^2}{2J^2}\right)\vec{J} \qquad (\text{now along } \vec{J}) \quad (42)$$

From Eq. (41),

$$E^{(1)}_{\text{Zeeman}} \cong \left\langle n\,l(s)\,j\,m_j \left| \frac{e}{2m_e}\vec{B}_{\text{ext}} \cdot \left(1+\frac{\hat{J}^2-\hat{L}^2+\hat{S}^2}{2\hat{J}^2}\right)\hat{\vec{J}} \right| n\,l(s)\,j\,m_j \right\rangle$$

$$= \left\langle n\,l(s)\,j\,m_j \left| \underbrace{\left(1+\frac{j(j+1)-l(l+1)+s(s+1)}{2j(j+1)}\right)}_{\text{just a number of order 1 (Landé g-factor)}} \frac{e}{2m_e}\hat{J}\cdot\vec{B}_{\text{ext}} \right| n\,l(s)\,j\,m_j \right\rangle$$

$$= \left\langle n\,l(s)\,j\,m_j \left| g_{\text{Landé}}\frac{e}{2m_e}\hat{\vec{J}}\cdot\vec{B}_{\text{ext}} \right| n\,l(s)\,j\,m_j \right\rangle \quad (43)$$

$\left(\vec{B}_{\text{ext}} = B_{\text{ext}}\hat{z}\right)$

[no lost of generality]

$$= \left\langle n\,l(s)\,j\,m_j \left| g_{\text{Landé}}\frac{e}{2m_e}\hat{J}_z B_{\text{ext}} \right| n\,l(s)\,j\,m_j \right\rangle$$

$$= g_{\text{Landé}}\frac{e\hbar}{2m_e} m_j B_{\text{ext}} = g_{\text{Landé}}\mu_B B_{\text{ext}} m_j \qquad (\text{Done!}) \quad (44)$$

- Inspect Eq.(37), it is equivalent to having approximated $\vec{\mu}_{total}$ by

$$\vec{\mu}_{J(approx)} = g_{Landé}\left(\frac{-e}{2m_e}\right)\vec{J} \qquad (45)$$

[retained standard form]

with $g_{Landé} \equiv 1 + \dfrac{j(j+1) - \ell(\ell+1) + s(s+1)}{2j(j+1)}$ (for the state under consideration)

$[\vec{J} = \vec{L} + \vec{S}]$ ($j$) ($\ell$) ($s$) all in!

- Retained the standard form between $\vec{\mu}$ and AM, and put all the complications into $g_{Landé}$, which can be determined experimentally

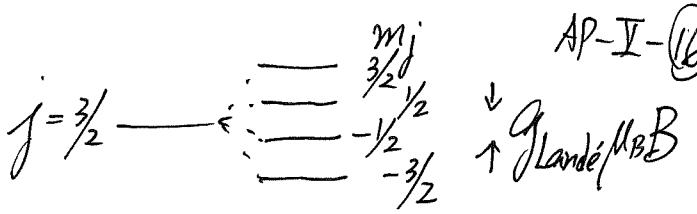
- Eq.(44): $E^{(1)}_{Zeeman} = \underbrace{g_{Landé}\,\mu_B B m_j}$ vs $\underbrace{g\,\mu_B B m_\ell}$ for "Normal Zeeman effect" due to $\vec{\mu}_L$ only

↖ same form ↗

$g_{Landé}$ vs $g$

$m_j$ vs $m_\ell$

$$\therefore\ E^{(1)}_{Zeeman} = g_{Landé}\,\mu_B\,B\,m_j$$




removes degeneracy due to $m_j$

**Example**

$\ell = 1$ (p states)

(6 states with spin)

Ignoring everything ($\hat{H}_{atom}$ only) + Including $\hat{H}'_{so}$ only + Weak field with $\hat{H}'_z$

**Weak Field**

$n$ p states (e.g. 2p) $-\frac{13.6eV}{n^2}$ (6 states) (H-atom)

(4 states) $j = 3/2$ — $m_j$: $+3/2$, $+1/2$, $-1/2$, $-3/2$

(2 states) $j = 1/2$ — $m_j$: $+1/2$, $-1/2$

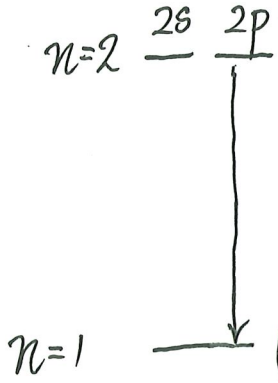
With $\hat{H}'_{so}$ only, energy depends on $n$ and $j$ (see fine-structure discussion)

[no external field here, $B_{ext} = 0$]

$B_{ext}$ lifts degeneracy as $E^{(1)}_{Zeeman} = g_{Landé}\mu_B m_j B_{ext}$ depending on $m_j$

# Consequences : Look into the first Lyman line (Weak Field)

$\hat{H}_{atom}$ only




$\rightsquigarrow \hbar\omega = \Delta E = -\frac{13.6}{4} - \left(-\frac{13.6}{1}\right) \approx 10.2\,eV$

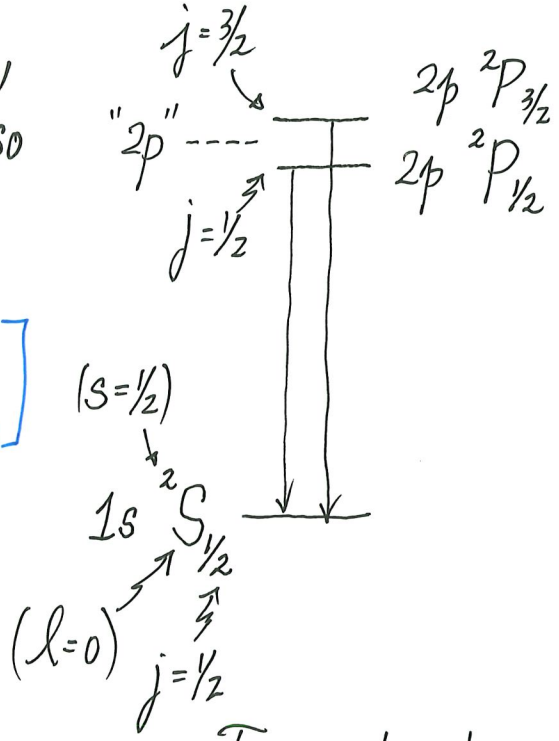
spectrum

one line at ~10.2eV (in Lyman Series)

$\hat{H}_{atom} + \hat{H}'_{so}$

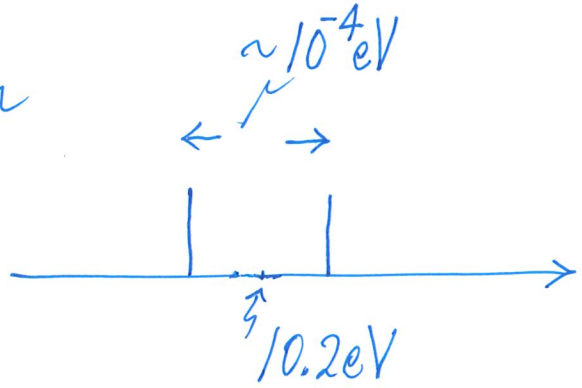
$(\vec{B}_{ext} = 0)$

[spin-orbit coupling only]




Fine structure

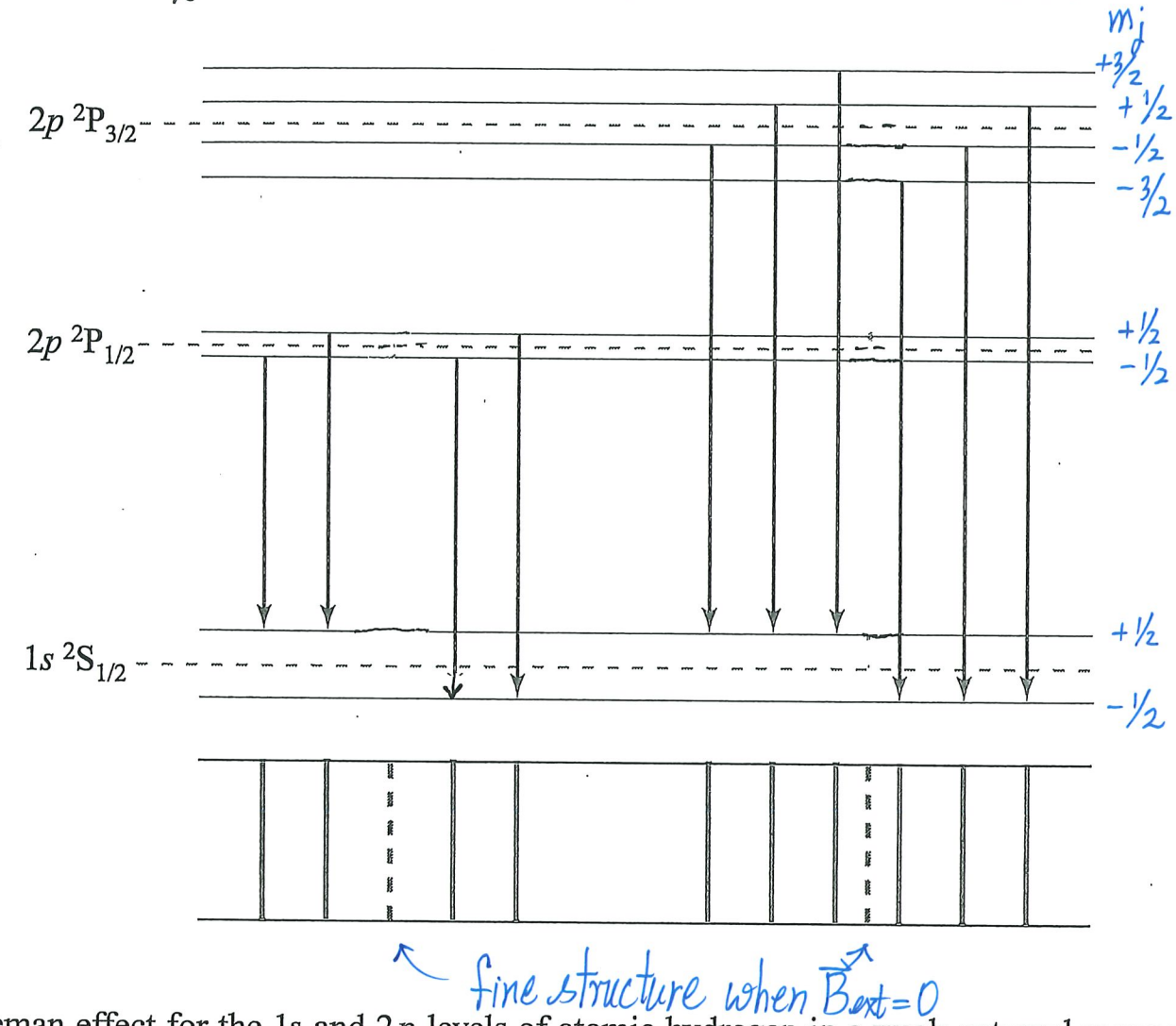
spectrum




due to spin-orbit interaction

$\hat{H}_{atom} + \hat{H}'_{so} + \hat{H}'_{z}$ ($\vec{B}_{ext} \neq 0$, but weaker than $\vec{B}_{int}$ in spin-orbit interaction)

Hydrogen
($n=2 \rightarrow n=1$)




fine structure when $\vec{B}_{ext}=0$

Selection rules for allowed transitions

$\Delta m_j = 0, \pm 1$

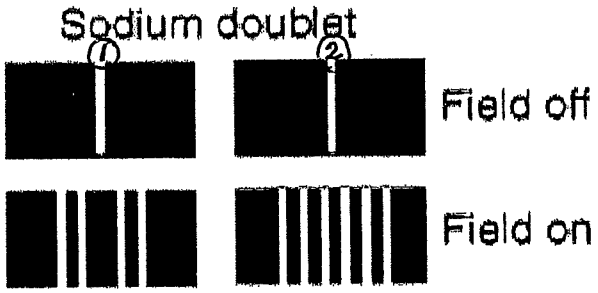
10 lines in weak $\vec{B}_{ext}$

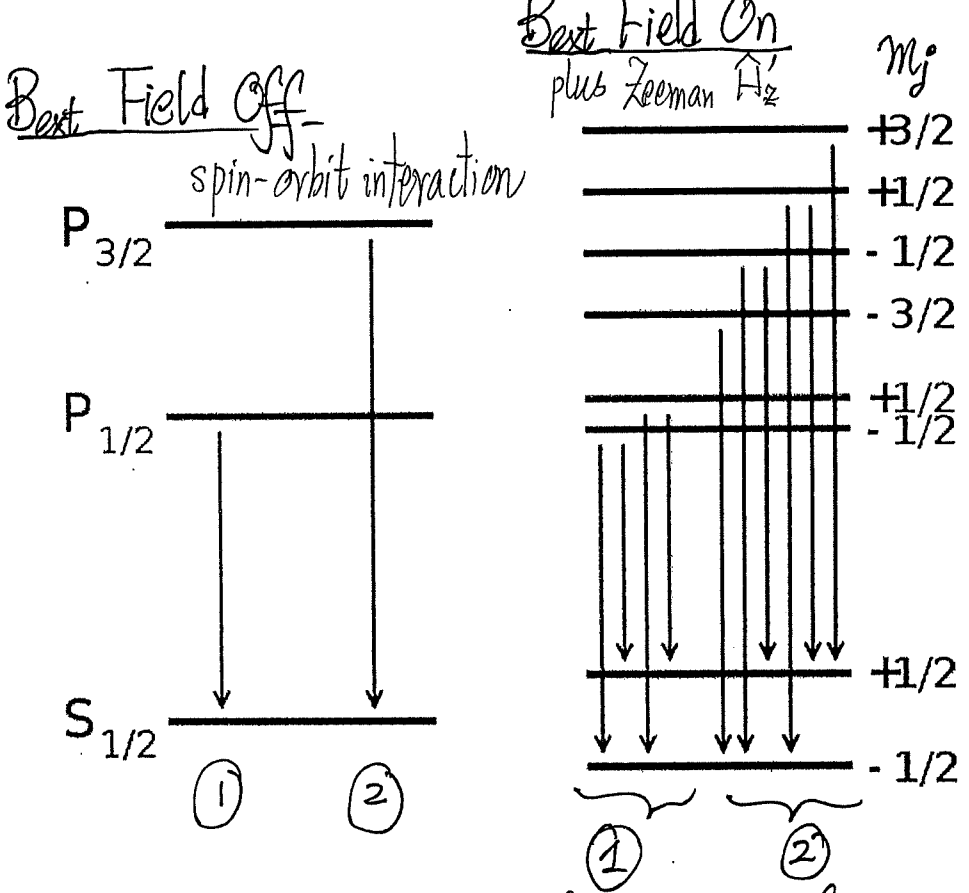
Anomalous Zeeman Effect

The Zeeman effect for the 1*s* and 2*p* levels of atomic hydrogen in a weak external magnetic field, showing the allowed transitions. A schematic diagram of the resulting spectrum is shown at the bottom. The dashed lines show the fine structure that is present in the absence of an external magnetic field.

## Same Works for Sodium

- $\hat{H}'_{so}$ only
  Sodium D lines (doublet)
- With $\vec{B}_{ext}$, each line in doublet split
- This example uses Na atom
- Bigger atoms have stronger $\hat{H}'_{so}$
- Thus, weak $\vec{B}_{ext}$ case is often observed in heavier atoms




Zeeman term leads to $g_{Lande}\,\mu_B\,m_j\,B_{ext}$

$\Delta m_j = 0, \pm 1$ selection rules

# Final Remarks

- The study on Zeeman effect had led to key developments in ideas/concepts in Quantum Mechanics
- QM and spin, plus techniques (approximations), are key to understand Fine structure and Zeeman effect
- Ideas: $\vec{J}$, spin-orbit interaction $\hat{H}'_{so}$, $\hat{H}_z$, strong/weak field cases...
  are introduced within the context of hydrogen atom here.
  But ideas/techniques are applicable to other atoms
  [need to add up $\vec{L}$'s and $\vec{S}$'s of many electrons]
- Historically, $g_{\text{Landé}}$ was introduced (1921) to describe exp'tal data before QM and spin were established. Its value informs us how important spin-orbit interaction is in an atom.

An appreciation...

Modern/Quantum Physics level : Descriptive
- Zeeman : "$-\vec{\mu}_L \cdot \vec{B}$"
- Spin-orbit: "$\vec{S} \cdot \vec{L}$"
- Hydrogen's relative correction: "ignored"
- Strong & Weak Field Zeeman: "not treated"

Applied Quantum Mechanics:

A bit of Quantum Mechanics (TISE, Angular Momenta in QM, Think classical and go quantum (look for $\hat{H}'_{so}$ and $\hat{H}'_z$)) and approximation methods (perturbation theory, physical sense) can take us to a deeper and quantitative understanding that backs up the qualitative descriptions.